Correspondence

The Characteristic Impedance of Square Coaxial Line

In a recent communication, Green [1] has given approximate results for the characteristic impedance of a coaxial line with square section inner and outer conductors. It is the purpose of this communication to point out that Bowman [2] has given an exact solution in terms of elliptic integrals, which is by no means computationally tedious. It is, in Green's notation

$$Z_0 = 5\pi v \times 10^{-8} K'(k)/K(k) \qquad (1)$$

where the modulus $k=(\lambda'-\lambda)^2/(\lambda'+\lambda)^2$ and λ and $\lambda'=(1-\lambda^2)^{1/2}$ are given implicitly in terms of the line dimensions by

$$K'(\lambda)/K(\lambda) = (b+a)/(b-a). \qquad (2)$$

The calculations may be carried out using tables of elliptic integrals, or by introducing the modular constant

$$q = \exp(-\pi K'/K)$$

and using the rapidly convergent theta function series [3]

$$\sqrt{k'} = \frac{1-2q+2q^4-2q^9\cdots}{1+2q+2q^4+2q^9\cdots}$$

and

$$q = \epsilon + 2\epsilon^5 + 15\epsilon^9 \cdots$$

where

$$2\epsilon = \frac{1-\sqrt{k'}}{1+\sqrt{k'}}$$

to find q from k' and vice versa. Tables of K'/K against values of k^2 have also been published [4].

Eqs. (1) and (2) may equally well be used to determine the line dimensions for a given characteristic impedance. For example, taking $v=2.9979\times 10^8$ m/sec, we find that for $Z_0=50\ \Omega$, these are given by $b/a=2.5076$, which is 0.14 per cent below the value in Green's Table II.

When b/a is a simple fraction, the modulus λ in (2) can be determined algebraically [3] from the theory of modular transformations, often with some simplification of the work. Thus, in the case $b/a=3$ discussed by Green, we have $K'/K=2$ and $\lambda=(\sqrt{2}-1)^2$, and the characteristic impedance is 60.6106 Ω, which is 0.14 per cent above the value in Green's Table I.

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A Varactor Tuned UHF Coaxial Filter

The purpose of the following communication is to present a method for designing a varactor-tuned UHF coaxial filter.[1] The method includes the means for determining the varactor diode characteristics necessary for a given tuning range and bandwidth.

The basic idea of the filter consists of an LC circuit where L is realized by a short-circuited coaxial line, and C is realized by the sum of the junction capacitance and cartridge capacitance of the varactor diode. It is desirable to use a diode in which the cartridge capacitance is small compared to the junction capacitance in order to maximize the contribution of the variable junction capacitance.

The geometry of a practical half-wave filter is shown in Fig. 1. Because of the balanced geometry of the half-wave resonator, an antinode of current exists at the diode position which will minimize the signal voltage drop across the diode. The button capacitor at one end of the resonator provides an RF short and dc isolation for the diode biasing potential. The diode biasing potential is applied between the center terminal of the button capacitor and the outer coaxial conductor. Furthermore, the use of diode holders (adapters) has the important advantage that they can be soldered to the two parts of the inner conductor while demounted from the diode cartridge.

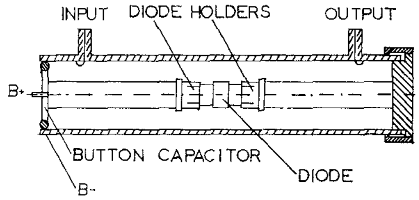

Fig. 1—The coaxial filter.

The equivalent circuit of the resonator is shown in Fig. 2. The diode impedance is closely given by

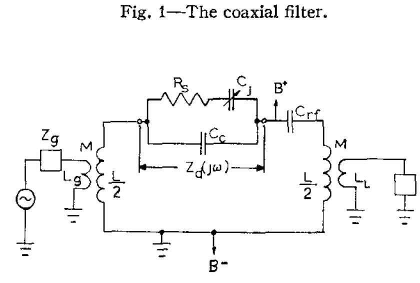

Fig. 2—The resonator equivalent circuit.

$$Z_d = r_s + 1/j\omega C_t \qquad (1)$$

when $\omega^2 R_s C_c C_j \ll 1$ and $(\omega R_s C_c C_j/C_t)^2 \ll 1$, where

$$c_t = C_c + C_j \qquad (2)$$

$$r_s = R_s(C_j/C_t)^2. \qquad (3)$$

For the unloaded resonator, at resonance ($\omega=\omega_0$) the magnitude of the capacitive reactance is equal to that of the total inductance; hence, at resonance,

$$\frac{1}{2C_t} = \omega_0 R_0 \tan\frac{\omega_0 l}{v} \equiv \omega_0 Z \qquad (4)$$

where R_0, l, and v are the characteristic impedance of the coaxial line comprising the resonator, the half length of the resonator center conductor, and the velocity of propagation within the resonator, respectively.

In regard to the length l, a suitable value is one half of the physical length of the inside of the resonator. Using such a value imposes a slight approximation because the ceramic diode cartridge makes the overall center conductor electrical length slightly longer than the geometric length. Furthermore, the resonator length reduction due to an increase in the dielectric constant is partially cancelled due to the curvature of the electric field lines at the middle of the resonator. Also, experiment has shown that it is an excellent approximation to consider the geometric half length of the inside of the resonator as the length l.

When the coupled reactive component is negligible compared to $\omega_0 L$, the reciprocal of the loaded Q can be expressed as

$$\frac{1}{Q_L} = \frac{r_s}{\omega_0 L} + \frac{2\omega_0^2 M^2 Z_0}{\omega_0 L(Z_0^2 + \omega_0^2 L_0)} \qquad (5)$$

where the first and second terms are the reciprocals of the unloaded resonator Q and the external Q component, Q_u and Q_e, respectively. The insertion ratio of the filter is given by

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \left(\frac{r_s Q_u}{|Z_t|\,Q_e}\right)^2 \qquad (6)$$

where

$$Z_t = r_s\left[1 + \frac{Q_u}{Q_e}\left(1 - j\frac{\omega_0 L_0}{Z_0}\right)\right]. \qquad (7)$$

From (4), two design curves can be derived. The first one, Fig. 3, gives the values of ωZ as a function of f with l as a parameter. The second one, Fig. 4, gives $\omega Z = \frac{1}{2}C_t$ as a function of the varactor bias voltage V_b with the zero bias junction capacitance C_{j0} as a parameter. The latter curve is based upon the assumption that the junction capacitance is accurately given by

$$C_j(V_b) = C_{j0}\left(1 - \frac{V_b}{V_0}\right)^{-1/k} \qquad (8)$$

where C_{j0}, V_0, and k are constants. Numerical values of these constants for a given diode are usually contained in the manufacturer's description of the diode; if their values are not given, they can be derived from a plot of $C_j(V_b)$ vs V_b. Similarly, by virtue of (3), a third design curve, Fig. 5,

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[1] A. P. Benguerel, "Coaxial Filter Tuned with a Varactor Diode," M.Sc. thesis, Dept. of Electrical Engineering (ERL), University of Kansas, Lawrence, Kan.: 1962